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Analysis of combined fully-developed natural convection heat and mass transfer between two inclined parallel plates

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Abstract—Fully-developed, combined, natural convection heat and mass transfer in an inclined open-ended channel is analyzed. General solutions to the governing equations are given and the conditions of existence of solutions are determined for all Lewis numbers. The results include expressions for the flow rate, Nusselt number and Sherwood number. Two examples illustrating the use of the general solutions are also presented.

1. INTRODUCTION

COMBINED natural convection heat and mass transfer processes have received considerable attention due to the importance of these phenomena in many industrial and natural processes. One particular application is open-cycle desiccant cooling systems [1]. The motion of the fluid is caused by the buoyancy forces arising from temperature and concentration gradients. The transport of energy and mass are governed by the equations of conservation of energy and mass for multicomponent mixtures. These equations are usually simplified by the following classical assumptions: only binary systems are considered; the Boussinesq approximation is valid; the concentration of the diffusing component is small (dilute systems); viscous dissipation, thermal-diffusion and diffusion-thermo effects are neglected and the two components of the mixture have essentially the same specific heats.

Using these assumptions the equations governing these processes can be simplified to

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{D\tau} = -\frac{\nabla \mathbf{p}}{\rho} + \nu\nabla^2\mathbf{u} + [1-\beta(t-t_0)-\beta^*(w-w_0)]\mathbf{g} \quad (2)$$

$$\frac{Dt}{D\tau} = \alpha\nabla^2 t \quad (3)$$

$$\frac{Dw}{D\tau} = D\nabla^2 w. \quad (4)$$

In natural convection with combined heat and mass transfer, most previous literature has concentrated on external flows [2–4] which allow similarity solutions for uniform wall conditions. Also, an analogy with heat transfer has been used when $Le = 1$ [2].

The vertical open-channel geometry has been extensively studied for the case of natural convection heat transfer [5–13]. Also, extensive numerical computations and experimental measurements have been done by Azevedo and Sparrow, to study the heat transfer characteristics of a developing natural convection flow field in an inclined channel [14]. However, limited work has been done on the simultaneous transport of heat and mass transfer in open-ended vertical and inclined channels [15–17].

Aung [10] obtained a closed-form solution for the case of heat transfer only, considering the flow to be fully developed. Following his analysis, Nelson and Wood [17] considered combined heat and mass transfer. The work presented in this paper is an extension and generalization of refs. [10, 17] where the general solvability conditions for all Lewis numbers are determined. The analysis also takes into account the effect of the inlet pressure and gives the dependence of the transfer coefficients on the channel length. The solution of the flow field and the heat and mass transfer rates, as presented in this paper, are also useful as means to check the accuracy of numerical solutions for combined heat and mass transfer at small Rayleigh numbers and would give an appropriate starting iteration for transient calculations.

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NOMENCLATURE

A_1, A_2 constants defined by equations (54) and (55)
b channel width
B_1, B_2 constants used in equation (44)
C_1, C_2 constants defined by equations (62) and (63)
D mass diffusivity
g acceleration due to gravity
Gr Grashof number, $(\Delta t)_r B^4 g \cos\phi/(l\nu^2)$
h_t heat transfer coefficient
h_w mass transfer coefficient
H dimensionless heat added to the fluid, $\int_0^1 U(Y)T(Y)\,dY$
k thermal conductivity
l channel length
L dimensionless channel length
Le Lewis number
m mass flux
M dimensionless mass added to the fluid, $\int_0^1 U(Y)W(Y)\,dY$
N ratio β^*/β
Nu_b Nusselt number based on b
p pressure
p_0 local hydrostatic pressure
P dimensionless pressure
Pr Prandtl number
q heat flux
Q dimensionless flow rate, $\int_0^1 U(Y)\,dY$
R constant defined by equation (23)
Ra Rayleigh number
s variable of integration
S constant defined in equation (43)
Sc Schmidt number
Sh_b Sherwood number based on b
t fluid temperature
T dimensionless fluid temperature
u velocity vector
U dimensionless longitudinal velocity
w mass fraction of the diffusing component
W dimensionless mass fraction
X dimensionless longitudinal coordinate
Y dimensionless coordinate across the channel.

Greek symbols
α thermal diffusivity
α_t, α_w constants defined in Section 3.2
β volumetric coefficient of thermal expansion
β^* volumetric coefficient of mass fraction expansion
δ_1, δ_2 constants equal to $P'''(X)$ in Section 2.1 and $P''(X)$ in Section 2.2, respectively
γ constant defined by equation (36)
ν kinematic viscosity
ρ fluid density
Φ angle of inclination.

Subscripts
0 ambient or at channel inlet
r reference quantity
s wall surface
t temperature
w mass fraction.

2. ANALYSIS

2.1. *Solvability conditions*

Considering the channel inclined at an arbitrary angle Φ, as shown in Fig. 1, the equations of change given above are greatly simplified by considering the flow to be fully developed. The dimensionless form of the governing equations become

$$P'(X) - U'''(Y) = T(X, Y) + NW(X, Y) \quad (5)$$

$$Pr\, U(Y)(\partial T/\partial X) = (\partial^2 T/\partial Y^2) \quad (6)$$

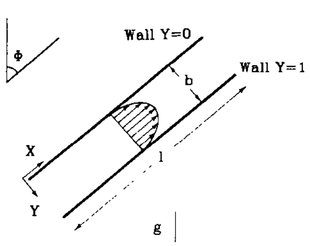

FIG. 1. Flow geometry and coordinate system.

$$Sc\, U(Y)(\partial W/\partial X) = (\partial^2 W/\partial Y^2) \quad (7)$$

where

$$X = x/(l\,Gr)$$
$$Y = y/b$$
$$U = ub^2/(l\,Gr)\nu$$
$$P = (p-p_0)b^4/(\rho l^2 \nu^2 Gr^2)$$
$$T = (t-t_0)/(\Delta t)_r$$
$$W = (w-w_0)/(\Delta w)_r$$
$$Gr = (\Delta t)_r b^4 \beta g \cos\Phi/(l\nu^2)$$
$$N = \beta^*(\Delta w)_r/\beta(\Delta t)_r.$$

Since the flow field is considered to be fully developed, the variation of inclination only affects the Grashof number by the cosine of the angle of tilt. The reference temperature and mass fraction differences $(\Delta t)_r$ and $(\Delta w)_r$ are chosen so as to obtain a convenient form for the boundary conditions. Differentiating equation (5) with respect to X and Y gives

$$P''(X) = (\partial T/\partial X)+N(\partial W/\partial X) \quad (8)$$

$$-U'''(Y) = (\partial T/\partial Y)+N(\partial W/\partial Y). \quad (9)$$

Combining equations (6)–(8) yields

$$P''(X)U(Y) = (1/Pr)(\partial^2 T/\partial Y^2) + (N/Sc)(\partial^2 W/\partial Y^2). \quad (10)$$

Integrating equation (10) twice with respect to Y gives

$$P''(X)\int_0^Y\left[\int_0^s U(z)\,dz\right]ds+f(X)Y+g(X) = (1/Pr)T(X,Y)+(N/Sc)W(X,Y) \quad (11)$$

where $f(X)$ and $g(X)$ are functions of integration determined by the boundary conditions at $Y = 0$ and 1. Since the same functions should be obtained, the following conditions of compatibility are required for the existence of a solution:

$$f(X) = [(1/Pr)(\partial T/\partial Y)+(N/Sc)(\partial W/\partial Y)]_{Y=0} = -P''(X)Q+[(1/Pr)(\partial T/\partial Y) + (N/Sc)(\partial W/\partial Y)]_{Y=1} \quad (12)$$

$$g(X) = (1/Pr)T(X,0)+(N/Sc)W(X,0) = P''(X)\int_0^1\left[\int_0^s U(z)\,dz\right]ds-f(X) + (1/Pr)T(X,1)+(N/Sc)W(X,1). \quad (13)$$

The conditions given by equations (12) and (13) are not sufficient and further restrictions apply on the prescribed quantities at the walls. To determine the allowable dependence of the boundary conditions with x, two cases of $Le \neq 1$ and $Le = 1$ need be considered separately.

2.1.1. *The case of* $Le \neq 1$. In this case the discriminant of the set of equations (5) and (11) is not equal to zero, and the following expressions are obtained for the temperature and mass fraction:

$$T(X,Y) = \frac{1}{Le-1}\left\{Sc\left[P''(X)\int_0^Y\left[\int_0^s U(z)\,dz\right]ds + f(X)Y+g(X)\right]-[P'(X)-U''(Y)]\right\} \quad (14)$$

$$W(X,Y) = \frac{1}{Le-1}\left\{Sc\left[P''(X)\int_0^Y\left[\int_0^s U(z)\,dz\right]ds + f(X)Y+g(X)\right]-Le[P'(X)-U''(Y]\right\}. \quad (15)$$

Forming $(\partial T/\partial X)$ and $(\partial^2 T/\partial Y^2)$ using equation (14) and substituting these expressions in equation (6) yields

$$Sc\left[P'''(X)\int_0^Y\left[\int_0^s U(z)\,dz\right]ds+f'(X)Y+g'(X)\right] -(1+Le)P''(X) = (1/Pr)\frac{U''''(Y)}{U(Y)}. \quad (16)$$

Taking the derivative of equation (16) twice with respect to Y yields

$$Sc\,Pr\,P'''(X)U(Y) = [U''''(Y)/U(Y)]''. \quad (17)$$

Equation (17) is valid only if $P'''(X)$ is equal to a constant, for convenience called δ_1. Differentiating equation (16) with respect to X yields

$$Sc\,[f''(X)Y+g''(X)]-(1+Le)\delta_1 = 0. \quad (18)$$

Equation (18) implies that $f''(X) = 0$ and $g''(X) = (Sc^{-1}+Pr^{-1})\delta_1$. Thus, $P(X)$, $g(X)$ and $f(X)$ are at most polynomials of degree 3, 2 and 1, respectively. Consequently, the prescribed temperature and mass fraction at the walls can be at most second-order polynomials of x. Also, the prescribed fluxes of heat and mass at the walls can be at most first-order polynomials of x. In addition, these boundary conditions should satisfy the compatibility relations given in equations (12) and (13).

2.1.2. *The case of* $Le = 1$. In this case the discriminant of the set of equations (5) and (11) is equal to zero. Combining these two equations gives

$$P''(X)\int_0^Y\left[\int_0^s U(z)\,dz\right]ds+f(X)Y+g(X) = (1/Pr)[P'(X)-U''(Y)]. \quad (19)$$

Differentiating equation (19) twice with respect to Y yields

$$U''''(Y)+Pr\,P''(X)U(Y) = 0. \quad (20)$$

This result implies that $P''(X)$ is equal to a constant, for convenience called δ_2. By forming the derivative of equation (19) with respect to X, we obtain

$$f'(X)Y+g'(X) = \delta_2/Pr \quad (21)$$

which shows that $f'(X) = 0$ and $g'(X) = \delta_2/Pr$. Therefore, $P(X)$, $g(X)$ and $f(X)$ are at most polynomials of degree 2, 1 and 0, respectively. Therefore, the prescribed fluxes of heat and mass at the walls can be at most constants satisfying the condition of equation (12). Also, the prescribed temperature and mass fraction at the walls can be at most first-order polynomials satisfying the condition given by equation (13).

3. APPLICATIONS OF THE ABOVE RESULTS

3.1. *Uniform temperature and mass fraction at the upper surface and uniform heat and mass flux into the fluid at the lower surface*

First consider the case where the boundary conditions are uniform temperature and mass fraction at the upper wall, $Y = 0$, and uniform heat and mass fluxes at the lower wall, $Y = 1$. Let the temperature and mass fraction at the upper wall be t_s and w_s, respectively, and let the flux of heat and mass from the lower wall into the fluid be q_s and m_s, respectively. Defining the quantities $(\Delta t)_r$, $(\Delta w)_r$, r_t and r_w as

$$(\Delta t)_r = (t_s - t_0), \quad (\Delta w)_r = (w_s - w_0)$$

$$r_t = q_s b/[k(\Delta t)_r], \quad r_w = m_s b/[\rho D(\Delta w)_r].$$

The dimensionless boundary conditions become

$$\text{for } P, \quad P(0) = -Q^2/2, \quad P(L) = 0$$

$$\text{for } T, \quad T(X,0) = 1, \quad (\partial T/\partial Y)_{Y=1} = r_t$$

$$\text{for } W, \quad W(X,0) = 1, \quad (\partial W/\partial Y)_{Y=1} = r_w.$$

Equation (5) implies that $P'(X)$ is constant. Solving for $U(Y)$ yields

$$U(Y) = \frac{1+N}{6}\left[3\left(1 - \frac{Q^2}{2L(1+N)}\right) + R(Y+1)\right]Y(1-Y) \quad (22)$$

where R is defined by

$$R = (r_t + Nr_w)/(1+N). \quad (23)$$

For the boundary conditions at the upper and lower walls considered in this example, the compatibility conditions are satisfied for all Lewis numbers. These conditions give

$$f(X) = (1/Sc)\,(r_t + Nr_w) \quad (24)$$

$$g(X) = (1/Sc)\,(1+N). \quad (25)$$

Substituting equations (24) and (25) in equations (14) and (15) yields

$$T(X, Y) = 1 + r_t Y \quad (26)$$

$$W(X, Y) = 1 + r_w Y. \quad (27)$$

Similar results were obtained by Nelson and Wood [17] who specifically considered the case of $Le = 1$ and arbitrarily generalized their results for all values of Le.

Integrating the velocity across the channel yields the expression of the dimensionless flow rate

$$Q = \int_0^1 U(Y)\,\mathrm{d}Y = Q_\infty\left(1 - \frac{Q^2}{24LQ_\infty}\right) \quad (28)$$

where Q_∞ is the limit of Q as L approaches infinity and is given by

$$Q_\infty = (1/24)\,(1+N)\,(2+R) \quad (29)$$

and an approximation of Q for small values of Gr is given by

$$Q = Q_\infty[1 - (1/24)Q_\infty\,Gr]. \quad (30)$$

The total heat added to the fluid is given by

$$H = H_\infty\left[1 - \frac{(2+r_t)Q^2}{48LH_\infty}\right] \quad (31)$$

where H_∞ is the limit of H as L approaches infinity and is given by

$$H_\infty = \frac{(1+N)}{24}\left[2 + R + r_t\left(1 + \frac{4}{15}R\right)\right]. \quad (32)$$

The average channel Nusselt number is

$$\overline{Nu_b} \equiv \frac{q_{\text{added}}b}{k(\Delta t)_r} = Gr\,Pr\,H. \quad (33)$$

Similar expressions are obtained for the total mass transport of the fluid by substituting Sc for Pr and r_m for r_t in the above relationships. Therefore, the expression for Sherwood number is given as

$$\overline{Sh_b} \equiv \frac{m_{\text{added}}b}{\rho D(\Delta w)_r} = Gr\,Sc\,M. \quad (34)$$

Table 1 gives a comparison of equations (30) and (31) for $N = 0$ with the results obtained by Glover [13] for the case of developing natural convection using the same boundary conditions including only heat transfer effects. It is seen that the fully-developed solution compares well with the numerical results at small Ra. For the same value of Ra, the accuracy decreases as the asymmetry expressed by r_t increases.

The heat transfer results were also compared to the fully-developed results given by Bar-Cohen and Rohsenow [18]. This comparison, for $N = 0$ and $R = 0$, gave the same correlation for the Nusselt number.

3.2. *Linear temperature variation on the upper wall and uniform heat and mass flux into the fluid at the lower wall*

The reference temperature and mass fraction are defined by

$$(\Delta t)_r = [t(l,0) - t_0], \quad (\Delta w)_r = [w(l,0) - w_0].$$

The dimensionless boundary conditions are

for P,

$$P(0) = -Q^2/2, \quad P(L) = 0$$

for T,

$$T(X,0) = \alpha_t(X-L) + 1, \quad (\partial T/\partial Y)_{Y=1} = r_t$$

for W,

$$W(X,0) = \alpha_w(X-L) + 1, \quad (\partial W/\partial Y)_{Y=1} = r_m$$

where $\alpha_t = [t(l,\ 0) - t(0,\ 0)]/l$ and $\alpha_w = [w(l,\ 0) - w(0,\ 0)]/l$. These boundary conditions also satisfy the compatibility conditions for all Lewis numbers. The pressure distribution in this case is given as

$$P(X) = (1/2)\,(X-L)\,[\gamma X + (Q^2/L)] \quad (35)$$

where γ is defined as

$$\gamma = \alpha_t + N\alpha_w. \quad (36)$$

Table 1. Comparison of equations (30) and (31) with the numerical results of ref. [13]

		Flow rate, Q			Total heat added, H		
r_t	Ra	Equation (30)	Reference [13]	Percentage error	Equation (31)	Reference [13]	Percentage error
0	0.104	0.0832	0.0830	0.3	0.0832	0.0830	0.3
	1.089	0.0828	0.0800	3.6	0.0828	0.0800	3.6
	2.947	0.0821	0.0750	9.5	0.0821	0.0750	9.5
1.0	0.165	0.1248	0.1237	0.8	0.1886	0.1870	0.8
	4.275	0.1212	0.1000	21.2	0.1832	0.1507	21.5
5.0	0.066	0.2913	0.2887	0.8	1.0543	1.0454	0.8
	1.688	0.2835	0.2333	21.5	1.0272	0.8461	21.4
	4.978	0.2700	0.1750	54.3	0.9799	0.5907	65.8
10	0.037	0.4994	0.4950	0.9	3.1355	3.1090	0.8
	0.957	0.4865	0.4000	21.6	3.0679	2.5230	21.4
	2.789	0.4642	0.3000	54.7	2.9240	1.7807	64.2

From the compatibility conditions the following expressions are obtained:

$$f(X) = -\gamma Q + (1/Sc)\,(Le\, r_t + N r_w) \quad (37)$$

$$g(X) = (1/Sc)\,[(Le\,\alpha_t + N\alpha_w)\,(X-L) + Le + N]. \quad (38)$$

Substituting these results in equation (16) yields

$$U''''(Y) + cU(Y) = 0 \quad (39)$$

where

$$c = Pr\,\alpha_t + N\,Sc\,\alpha_w. \quad (40)$$

The velocity profile is obtained by solving equation (39), with the no-slip conditions $U(0) = U(1) = 0$ and the following conditions obtained by writing equations (1) and (5) at $Y = 0$ and 1, respectively:

$$-U''(0) = (1+N)S \quad (41)$$

$$-U''(1) = (1+N)R \quad (42)$$

where R is defined in equation (23) and S is given by

$$S = 1 - \frac{Q^2}{2L(N+1)} - \frac{\gamma L}{2(N+1)}. \quad (43)$$

The form of the solution depends on the sign of parameter c. Three cases of $c = 0$, $c < 0$ and $c > 0$ are considered separately. In each of these cases, the velocity, temperature and mass fraction profiles are determined and expressions for the transfer quantities are given.

3.2.1. *The case of* $c = 0$. Solving equation (39) for the velocity profile and using the no-slip conditions gives

$$U(Y) = (B_1 Y + B_2)\,Y(1-Y) \quad (44)$$

where B_1 and B_2 are determined using equations (41) and (42). For this case, the flow rate Q is given by

$$Q = (1/24)\,(1+N)\,(R+2S). \quad (45)$$

The profiles of temperature and mass fraction are obtained by substituting the expressions of $U(Y)$, $f(X)$, $g(X)$ and $P(X)$ into equations (14) and (15). We obtain

$$T(X, Y) = \alpha_t(X-L) + 1 + (r_t - Pr\,\alpha_t Q)\,Y - Pr\,\alpha_t F(Y) \quad (46)$$

$$W(X, Y) = \alpha_w(X-L) + 1 + (r_w - Sc\,\alpha_w Q)\,Y - Sc\,\alpha_w F(Y) \quad (47)$$

where

$$F(Y) = [(1+N)/120]\,[RY^2 + 5SY - (10/3)\,(R+3S)]\,Y^3. \quad (48)$$

The local Nusselt and Sherwood numbers at the walls are obtained by calculating the dimensionless heat and mass fluxes at each wall. These are given as

at $Y = 0$

$$Nu_{b,0}(X) \equiv \frac{h_t b}{k} = \frac{-r_t + Pr\,\alpha_t Q}{\alpha_t(X-L)+1} \quad (49)$$

$$Sh_{b,0}(X) \equiv \frac{h_w b}{D} = \frac{-r_w + Sc\,\alpha_w Q}{\alpha_w(X-L)+1}; \quad (50)$$

at $Y = 1$

$$Nu_{b,1}(X) = r_t \Big/ \left[\alpha_t(X-L) + 1 + r_t - \frac{1+N}{360} Pr\,\alpha_t(8R+3S)\right] \quad (51)$$

$$Sh_{b,1}(X) = r_w \Big/ \left[\alpha_w(X-L) + 1 + r_w - \frac{1+N}{360} Sc\,\alpha_w(8R+3S)\right]. \quad (52)$$

3.2.2. *The case of* $c < 0$. Using the same procedure followed in the previous section, the flow rate is given by

$$Q = (1/\lambda)\,(A_1 + A_2)\,[(1-\cos\lambda)\sinh\lambda + \sin\lambda\,(1-\cosh\lambda)] \quad (53)$$

$$A_1 = \frac{(1+N)S}{2\lambda^2 \sin\lambda \sinh\lambda} \quad (54)$$

$$A_2 = \frac{(1+N)}{\lambda^3 \sin\lambda \cosh\lambda + \cos\lambda \sinh\lambda} \times \left[R + \frac{\lambda S(\sin\lambda + \sinh\lambda)}{2\sin\lambda \sinh\lambda} \right] \quad (55)$$

which is an implicit equation for Q since A_1 and A_2 depend on Q. The profiles of temperature and mass fraction are given by

$$T(X, Y) = \alpha_t(X-L) + 1 + (r_t - Pr\,\alpha_t Q)Y - Pr\,\alpha_t F(Y) \quad (56)$$

$$W(X, Y) = \alpha_w(X-L) + 1 + (r_w - Sc\,\alpha_w Q)Y - Sc\,\alpha_w F(Y) \quad (57)$$

where

$$F(Y) = (1/\lambda^2)\{A_1[(1-Y)\sin\lambda \sinh\lambda + (1-Y)\sin\lambda \sinh\lambda - 2\sin\lambda \sinh\lambda] + A_2[Y\sin\lambda \sinh\lambda + \sin\lambda \sinh\lambda Y] - (1/\lambda^2)(1+N)RY\}. \quad (58)$$

By evaluating the heat and mass flux at $Y = 0$, we obtain the same expressions as given in equations (49) and (50). However, the expressions for Nusselt and Sherwood numbers at $Y = 1$ become

$$Nu_{b,1}(X) = r_t \Big/ \left\{ \alpha_t(X-L) + 1 + r_t - \frac{\alpha_t Pr}{\lambda^4}[2\lambda^2(A_2 - A_1)\sin\lambda \sinh\lambda - (1+N)R] \right\} \quad (59)$$

$$Sh_{b,1}(X) = r_w \Big/ \left\{ \alpha_w(X-L) + 1 + r_w - \frac{\alpha_w Sc}{\lambda^4}[2\lambda^2(A_2 - A_1)\sin\lambda \sinh\lambda - (1+N)R] \right\}. \quad (60)$$

3.2.3. *The case of $c > 0$.* In this case we obtain the non-dimensional flow rate which is given as

$$Q = (1/2\lambda)(\sinh\lambda - \sin\lambda)(C_1 + C_2) \quad (61)$$

where

$$C_1 = \frac{(1+N)}{2\lambda^2(\sin 2\lambda + \sinh 2\lambda)}[S\sin\lambda + (R/\lambda)\cosh\lambda] \quad (62)$$

$$C_2 = \frac{(1+N)}{2\lambda^2(\sin 2\lambda + \sinh 2\lambda)}[S\sinh\lambda + (R/\lambda)\cos\lambda] \quad (63)$$

which also is an implicit equation to solve for Q since C_1 and C_2 depend on Q. Expressing equation (61) in terms of a Taylor series expansion, we obtain the approximation

$$Q = \frac{1+N}{24}\left[2 + R - \frac{Q^2}{2(1+N)L} - AL + \frac{4\lambda^4}{15}\left(1 - \frac{Q^2}{2(1+N)L} - \frac{AL}{2} - \frac{71R}{144}\right) + O(\lambda^8) \right] \quad (64)$$

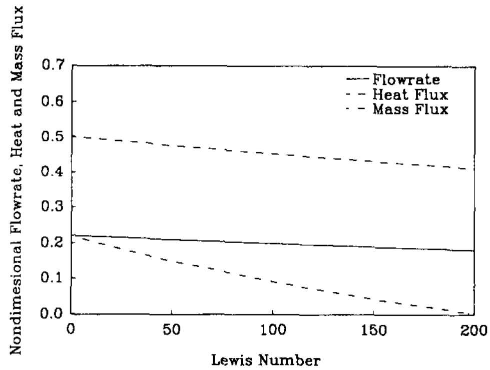

FIG. 2. Effect of Lewis number on the flow rate and the total heat and mass flux for fixed Prandtl number ($Pr = 0.7$, $Gr = 1.37$, $N = 0.5$, $\alpha_t = \alpha_w = 0.1$, $r_t = 2.5$ and $r_w = 0$).

where this equation reduces to equation (28) for $\lambda = 0$. The effect of Le appears only in the term of order λ^4 which explains the accuracy of the approximation given by Nelson and Wood [17]. The profiles of temperature and mass fraction are given by

$$T(X, Y) = \alpha_t(X-L) + 1 + r_t Y + \frac{\alpha_t}{\alpha_t + N\,Le\,\alpha_w}F(Y) \quad (65)$$

$$W(X, Y) = \alpha_w(X-L) + 1 + r_w Y + \frac{Le\,\alpha_w}{\alpha_t + N\,Le\,\alpha_w}F(Y) \quad (66)$$

where

$$F(Y) = 2\lambda^2[C_1(1-Y)\cos\lambda \cosh\lambda Y + C_2(1-Y)\cos\lambda Y \cosh\lambda] + (N+1)(RY+S). \quad (67)$$

Again evaluating the heat and mass flux at $Y = 0$ gives expressions analogous to equations (49) and (60). However, the expressions of Nu and Sh at $Y = 1$ become

$$Nu_{b,1}(X) = r_t \left\{ \alpha_t(X-L) + 1 + r_t + \frac{\alpha_t}{\alpha_t + N\,Le\,\alpha_w} \times [2\lambda^2(C_1 + C_2)(\cosh\lambda - \cos\lambda) - (r_t + Nr_w)] \right\}^{-1} \quad (68)$$

$$Sh_{b,1}(X) = r_w \left\{ \alpha_w(X-L) + 1 + r_w + \frac{Le\,\alpha_w}{\alpha_t + N\,Le\,\alpha_w} \times [2\lambda^2(C_1 + C_2)(\cosh\lambda - \cos\lambda) - (r_t + Nr_w)] \right\}^{-1}. \quad (69)$$

Figure 2 gives the variation of flow rate, Q, the total heat, H, and the total mass, M, added to the fluid with Le for the conditions

$$Gr = 1.367, \quad N = 0.5$$
$$r_t = 2.5, \quad r_w = 0$$
$$\alpha_t = 0.1, \quad \alpha_w = 0.1.$$

It can be seen that the flow rate, Q, the total heat, H, and mass, M, added to the fluid decrease with increasing Le for a fixed Pr. However, the reduction

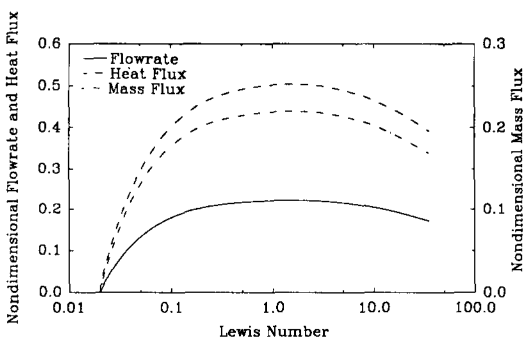

FIG. 3. Effect of Lewis number on the flow rate and the total heat and mass flux for fixed Schmidt number ($Sc = 0.7$, $Gr = 1.37$, $N = 0.5$, $\alpha_t = \alpha_u = 0.1$, $r_t = 2.5$ and $r_w = 0$).

in mass flux with increasing Lewis number is larger than the change in the volumetric flow rate and the heat flux. However, when Sc is fixed, the quantities Q, H and M increase, reaching a maximum value at about $Le = 1$ and decrease with increasing Le as shown in Fig. 3. These results indicate that the flow rate, and the total heat and the mass added to the fluid depend separately on the Schmidt and Prandtl numbers and not only on their ratio as given by Lewis number.

4. CONCLUSIONS

Combined natural convection heat and mass transfer between two inclined, parallel plates was studied for the case of fully-developed flow. Solvability conditions for which the governing equations have solutions are determined in terms of the allowable profiles for the conditions prescribed at the walls. When $Le = 1$, the prescribed temperature and mass fraction at the walls can be at most linear functions of the longitudinal coordinate and the fluxes of heat and mass prescribed at the walls should be constant. If $Le \neq 1$, the prescribed temperature and mass fraction at the walls can be at most second-order polynomials and the fluxes of heat and mass prescribed at the wall can be at most linear functions of the longitudinal coordinate.

Two cases illustrating the method were considered in detail and closed-form expressions were obtained for the profiles of velocity, temperature and mass fraction and the Nusselt and Sherwood numbers. The first case considered a uniform temperature and mass fraction was imposed on the upper wall and uniform fluxes of heat and mass were imposed on the lower wall. In this case, it was found that the profiles of velocity, temperature and mass fraction do not depend on Prandtl and Schmidt numbers. The second case considered a linear variation of temperature and mass fraction at the upper wall and uniform fluxes of heat and mass were imposed on the lower wall. In this case it was found that the profiles of velocity, temperature and mass fraction depend on Prandtl and Schmidt numbers and the other flow parameters. In particular, for a fixed value of Pr, the flow rate Q, the total heat H and the total mass M added to the fluid reach a maximum value at approximately $Le = 1$.

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ANALYSE DES TRANSFERTS COMBINES DE CHALEUR ET DE MASSE EN CONVECTION NATURELLE ETABLIE ENTRE DEUX PLAQUES PARALLELES INCLINEES

Résumé—On analyse les transferts simultanés de chaleur et de masse en convection naturelle pleinement établie dans un canal incliné ouvert. Des solutions générales des équations de base sont données et les conditions d'existence des solutions sont déterminées pour tous les nombres de Lewis. Les résultats incluent les expressions du débit, des nombres de Nusselt et de Sherwood. On présente aussi deux exemples illustrant l'utilisation de la solution générale.

UNTERSUCHUNG DES KOMBINIERTEN, VOLL AUSGEBILDETEN WÄRME- UND STOFFÜBERGANGS BEI NATÜRLICHER KONVEKTION ZWISCHEN ZWEI GENEIGTEN PARALLELEN PLATTEN

Zusammenfassung—Es wird der kombinierte Wärme- und Stoffübergang bei voll ausgebildeter natürlicher Konvektion in einem geneigten, an den Enden offenen Kanal untersucht. Allgemeine Lösungen der Erhaltungsgleichungen werden angegeben und die Bedingungen für die Existenz von Lösungen für alle Lewis-Zahlen ermittelt. Die Ergebnisse enthalten Ausdrücke für den Durchsatz, die Nusselt-Zahl und die Sherwood-Zahl. Weiterhin werden zwei Beispiele zur Anwendung der allgemeinen Lösungen angegeben.

АНАЛИЗ ЕСТЕСТВЕННОКОНВЕКТИВНОГО СЛОЖНОГО ПОЛНОСТЬЮ РАЗВИТОГО ТЕПЛО-И МАССОПЕРЕНОСА МЕЖДУ ДВУМЯ НАКЛОННЫМИ ПАРАЛЛЕЛЬНЫМИ ПЛАСТИНАМИ

Аннотация—Анализируется естественноконвективный полностью стабилизированный совместный тепло-и массоперенос в наклонном канале без торцевых стенок. Даны общие решения основных уравнений и определены условия их существования для всех значений числа Льюиса. Получены выражения для скорости течения, а также для чисел Нуссельта и Шервуда. На двух примерах показано использование общих решений.